British Contribution to Structural Response to Noise

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The normal mode theory formulated by Powell has formed the framework for extensive theoretical and experimental studies of the important parameters in the response of structures to noise. The excitation forces have been defined by their cross spectral density and the predominant mode shapes in typical structures have been identified. Reduction in the response levels has been achieved by the addition of damping. An estimation method has been evolved for use in design and comparisons made with a range of experimental results. Recent developments of the wave propagation method offer the possibility of a better understanding of structural response to traveling pressure fields.

Nomenclature

A = area
G = spectral density function
M = generalized mass
f = mode shape function
$\mathbf{x}$ = vector position on structural surface
y = displacement
t = time
η = loss factor (hysteretic damping)
μ = real or imaginary part of propagation constant
ω = natural frequency
σ = stress
ϵ = propagation constant

Subscripts

p = pressure
1,2,3 = positions on surface
r,s = mode numbers
r,i = real, imaginary
o = identifies static stress parameter

I. Introduction

THE British contributions to the study of the response of structures to noise have been primarily concerned with jet noise as the forcing function. Some work has also been done on boundary-layer pressure fluctuations and recently there has also been an interest in separated flow. The framework for these studies was first formulated by Powell[1] using the normal mode theory. This formulation had the advantage of showing clearly the important parameters of the problem. Subsequent work has concentrated on these constituent elements and recently design data sheets have been produced.

The normal mode formulation works well in practice if only a few modes (preferably lower-order modes) are predominant in the response. An evaluation of only two or three terms in the series is then often adequate to give a satisfactory approximation to the over-all response. However, in many practical cases such as stiffened skins and box structures, relatively high modal densities appear in the response. The further complication of modal overlap resulting from high damping has led to the development of a wave propagation solution by Mead. This recent development promises to extend the range of solutions available for practical problems. A start has also been made on a design data sheet version of this work.

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II. Normal Mode Theory

Powell showed that when a continuous structure, such as an aircraft fuselage or control surface, is excited by broad-frequency-band random forces, the resulting vibration can be treated as the summation of responses in a relatively large number of modes producing an overall picture of broad-band response. In some types of structure, a few or even a single mode may predominate but, in general, the broader band response situation must be considered.

Using the normal mode method, the spectral density of the displacement at any point $\mathbf{x}_1$ on the surface of the structure can be expressed in terms of the normal modes of the structure and the characteristics of the pressure field as

$$G(\mathbf{x}_1\omega) = \sum_{r=1}^{\infty}\sum_{s=1}^{\infty} \times \frac{f_r(\mathbf{x}_1)f_s(\mathbf{x}_1)\int_A\int_A f_r(\mathbf{x}_2)f_s(\mathbf{x}_3)G_p(\mathbf{x}_2\mathbf{x}_3\omega)dAdA}{M_r(\omega_r^2 - \omega^2 + in_r\omega_r^2)M_s(\omega_s^2 - \omega^2 - in_s\omega_s^2)} \quad (1)$$

Equation (1) shows that in addition to the normal modes, undamped natural frequencies, and modal damping factors it is also necessary to know the cross spectral density of the acoustic pressures.

A great simplification can be made if it is assumed that the space correlation coefficient is a Dirac delta function in space. The double area integration of Eq. (1) reduces to a single integration of the square of the mode shape which then cancels out with one of the generalized mass terms. The double series summation reduces to a single series summation. This assumption has been made in several theoretical studies of the response of structures to boundary layer pressure fluctuations. In most jet noise situations, however, the spatial scale of the pressure correlation coefficient is of the same order as the mode wavelength and thus cannot be treated as a delta function. In these cases there is coupling of the modal responses due to the pressure cross correlation term. In practice, however, the damping is usually very small and therefore the reciprocal of the frequency function in the denominator will only have nonzero values at frequencies close to the natural frequency of each mode. Thus the cross terms will be very small except when natural frequencies occur at separations smaller than the modal bandwidth.

An investigation of the effect of the cross terms in a lightly damped system has been made by Mercer[2] who considered the case of a continuous beam resting on many supports and excited by acoustic pressures. In the case showing maximum coupling the cross terms only contributed an additional 20% to the overall rms response level. In more typical cases, the contribution was of the order of

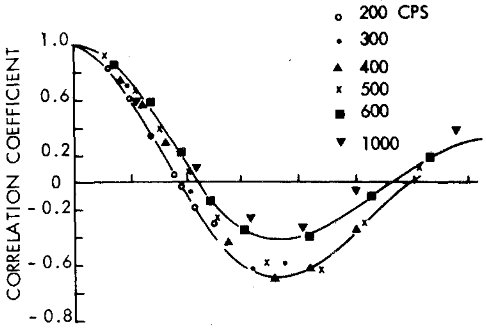

Fig. 1 Narrow band space correlation function, near field jet noise.

5-10%. Thus only relatively small errors are likely to arise from the neglect of the cross terms.

However, even if the cross terms are small, a large number of direct terms will be significant if the excitation forces cover a broad frequency range. Thus in cases of broad-band excitation the computation of response using Eq. (1) for all the modes having frequencies in the bandwidth of the excitation is prohibitively lengthy. Thus there is a great interest in the development of simplified models of the structure which lead to a tractable response equation.

If the cross terms are neglected the spectral density of displacement can be rewritten as

$$G(\mathbf{x}_1\omega) = \sum_{r=1}^{\infty} \times \frac{f_r^2(\mathbf{x}_1)\int_A\int_A f_r(\mathbf{x}_2)f_r(\mathbf{x}_3)G_p(\mathbf{x}_2\mathbf{x}_3\omega)dAdA}{M_r^2[(\omega_r^2 - \omega^2)^2 + \eta_r^2\omega_r^4]} \quad (2)$$

The mean-square value of the over-all displacement at $\mathbf{x}_1$ is obtained by integrating Eq. (2) over all frequencies. Thus

$$\overline{y^2(t)} = \int_0^\infty G(\mathbf{x}_1\omega)d\omega \quad (3)$$

If the modes are well separated in frequency, the damping is small, and the cross spectral density of pressure is changing very slowly with frequency then the frequency integration can be performed separately for each mode and Eq. (3) can then be written as

$$\overline{y^2(t)} = \sum_{r=1}^{N} \frac{\pi}{2\omega_r^3\eta_r} \frac{f_r^2(\mathbf{x}_1)}{M_r^2} \times \int_A\int_A f_r(\mathbf{x}_2)f_r(\mathbf{x}_3)G_p(\mathbf{x}_2\mathbf{x}_3\omega_r)dAdA \quad (4)$$

There has been considerable experimental and theoretical work on the individual components required for a solution of Eq. (4). In most areas contributions have been made in both the USA and Europe and good collaboration between the groups of workers existed. The individual components will now be discussed under separate headings.

III. Cross Spectral Density of Pressure

In general, the cross spectral density function G_p ($\mathbf{x}_2$, $\mathbf{x}_3$, ω) has to be defined for a very large number of pairs of points. This is clearly impracticable and it is therefore usual to make assumptions such as homogeneity of the pressure field. This is adequate for high-frequency vibrations which tend to be coherent over relatively small proportions of the total structural area. In this case, the cross spectral density function can be replaced by the product of the direct spectral density (now assumed to be constant over the structural area being considered) and the narrow-band cross-correlation function of the pressures at a range of separations of the two points.

Early theoretical work by Lighthill[3] and Lilley[4] led to an understanding of the way in which a turbulent mixing region of a jet exhaust could radiate sound into the far field. Further contributions were made by Ffowcs Williams[5] who looked at Mach wave radiation present in supersonic jets. The Lighthill theory was extended by Curle[6] to include a boundary and so enabled the theory of radiation of sound from a boundary layer to be developed. This work has been summarized in the Bakerian Lecture 1961.[7] These theoretical studies were complemented by experimental and semiempirical investigations such as those of Richards[8] and Powell.[9]

The majority of this work used 'far field' approximations and so was relevant to the community noise aspect. From the structural response point of view, the near field pressure of a jet exhaust or the wall pressure fluctuations in a boundary layer are required to provide the forcing functions. The pressures in these regions are much more influenced by the hydrodynamic details of the flow and therefore less amenable to the kind of dimensional analysis which was used to provide an insight into the far field radiation. For example, in the far field of a subsonic jet, the pressures are proportional to V^4, whereas in the flow itself, the hydrodynamic pressures are proportional to V^2. Measurements in the near field of the jet (up to a radius of about 4 diam) show that the velocity index varies from 2 to 4.

In the near field of jet flows, many measurements have been made in Europe[10-12] in order to derive empirical relationships for the over-all pressure and the pressure spectrum. The latest attempt to collate this information is the "Data Sheet on Estimation of Near Field Jet Noise Pressures."[13]

A set of definitive wall pressure measurements were made by Bull[14] in a subsonic boundary layer. These contributed to the experimental data on pressure spectra. Unfortunately the tunnel used had very limited supersonic capabilities and therefore these contributions must be regarded as being limited to subsonic flow.

Relatively little work has been done on the pressure correlation function. Following early work in the USA some measurements were made during two tests on structural response to jet noise and also some model experiments were carried out by Clarkson[15] to study the problems of scaling. A typical set of ⅓ octave band pressure correlation measurements close to a jet are shown in Fig. 1. In a subsonic boundary layer, some extensive measurements were made by Bull.[14] Figure 2 shows over-all longitudinal pressure correlations in a subsonic boundary layer. These can be used to make reasonable estimates of the correlation functions but because there is no theoretical work to support these empirical results there can be little confidence in extrapolating to other configurations.

IV. Response of Typical Structures to Random Pressure Fluctuations

In a high-intensity jet noise environment, the acoustic pressures on the structure are broad-band random in na-

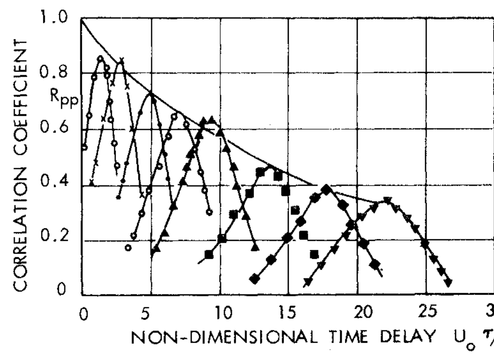

Fig. 2 Peaks of longitudinal space-time correlation of boundary-layer pressure fluctuations for a range of separations.

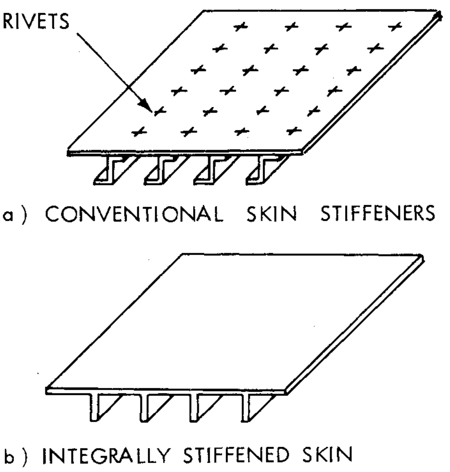

Fig. 3 Stiffened skin configurations.

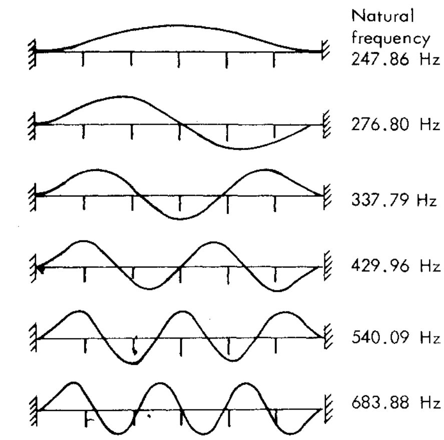

Fig. 5 Normal modes for an integrally stiffened plate.

ture covering a frequency range from below 100 to above 2000 Hz. A similar frequency range is associated with the pressure fluctuations in the turbulent boundary layer on a high-speed aircraft. As a normal mode analysis of an aircraft structure over this wide frequency range is clearly impractical, much early work in Europe was aimed at developing simpler structural models which could be used to compute the predominant features of the response spectra of typical structures. In this frequency range, the structural vibrations have been shown to be predominantly local in character and not involve over-all bending of wings or fuselage.[16] Full-scale measurements have shown that the predominant modes of vibration are those in which the major displacements are either in the skin or in the frame or rib support structure with wavelengths of the same order of magnitude as the stiffener spacing. The two main types of structure considered in this paper are stiffened skins and box structures with relatively light internal connecting members.

A. Response of Stiffened Plates

The two main types of stiffened plate construction are illustrated in Fig. 3. The stiffeners used in Fig. 3a may vary in cross section from the Z type shown, but in most cases they have a higher bending stiffness than the equivalent integral form shown in Fig 3b.

Built-up Stiffeners

In a complete structure the stiffened skin is usually supported on ribs or frames which are considerably stiffer than the stringers. The individual plates themselves usually have a high aspect ratio and thus the effect of the fixing along the short edges is small. The predominant panel vibration is then one in which adjacent plates are vibrating in association with the flexible stringers. The response of a conventional built-up structure of this type has been investigated by Clarkson[17] and Mercer and Seavey[18] have contributed to American work based on the transfer matrix method. The natural frequencies are grouped together in bands. The lowest band contains modes which have only one-half wavelength along the plate in the direction parallel to the stringers. The mode having the lowest natural frequency is usually one in which adjacent plates vibrate out of phase and the stringers twist (stringer twisting mode). The mode having the highest natural frequency in the first band is then one in which all plates vibrate in phase and the stringers bend (stringer bending mode). In between these two bounding modes there are intermediate modes in which the stringers both bend and twist. These effects are illustrated in Fig. 4. As the torsional stiffness of the stringer increases, the frequency of the lower bound will increase and the frequency bandwidth decrease.

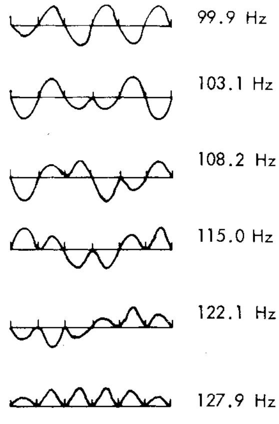

Fig. 4 Normal modes for a conventionally stiffened six bay plate.

Integral Stiffeners

In typical integrally stiffened skins, the stringers are of rectangular cross section and have different properties from the Z section and closed section stringers used in built-up designs. As a result of their very low bending stiffness, the mode shapes are no longer dominated by the position of the stiffeners and the lateral bending displacement of the stiffeners can be nearly as great as the lateral displacement of the centers of the skin plates.

The stringer torsion mode has a higher frequency than the stringer bending mode which is the reverse of the condition which holds for most built-up designs. A typical set of mode shapes is shown in Fig. 5.

B. Box-Type Structures

In many aircraft configurations the horizontal and vertical stabilizer structures are situated in regions of high noise levels from the jet effluxes. The tailplane and fin structure usually have stiffened skins to carry the over-all bending loads. The skins have interconnecting ribs (chordwise) and spars (spanwise). The ribs generally carry relatively small shear forces and so are thinner than the skin. They usually have lightening holes and often have stiffeners between the lightening holes.

When such a structure is excited by broadband pressures such as jet noise, a complicated form of vibration takes place. The skins vibrate as interconnected stiffened plates between the ribs, and the internal rib structure also vibrates. The response spectrum for each component usually shows several predominant peaks and although the

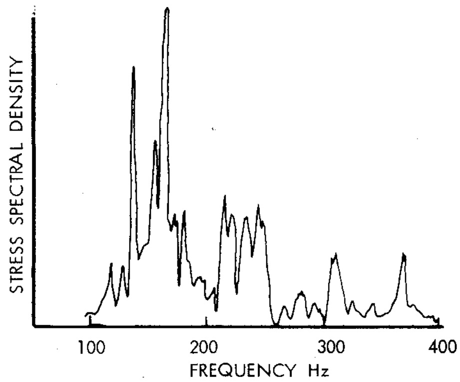

Fig. 6 Spectral density of stress in skin of multibay box structure.

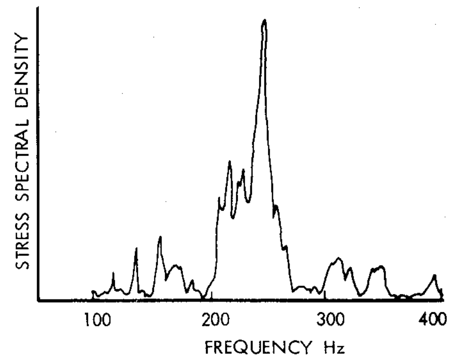

Fig. 7 Spectral density of stress in rib of multibay box structure.

skin must be forcing the ribs to vibrate, the peaks in the skin and rib spectra often do not coincide in frequency, as shown in Figs. 6 and 7. Naturally, there is a coincidence but the predominant peaks are not always the same in the two sections of the structure. When the excitation is applied to only one skin surface, the rib interconnection ensures that the other skin surface also vibrates. The vibration level in the skin which is not directly excited is usually about equal to the level in the other skin.

V. Computer-Based Methods for Natural Frequencies and Normal Modes

The normal modes and natural frequencies of stiffened skin structures have been computed using the Transfer Matrix method as mentioned in Sec. IV-A. An alternative exact solution has been developed recently by Wittrick and Williams.[19] This is an extension of their previous work on buckling of stiffened plates.

The above analytical methods are restricted to relatively simple assemblies of plates. To analyse more general configurations, finite element methods are being developed in several countries. Up to the present time, the majority of the work has been on static analysis or dynamic analysis in the low-frequency over-all mode regime. Some workers are now applying these methods to the short-wavelength modes of vibration in the audio-frequency range.[20] In this method the skins, stiffeners, and ribs can be represented by a series of interconnected rectangular or triangular plate elements. Approximate forms for the dynamic displacement of the element are assumed and the computer is used to assemble the elements in a compatible way. Beam elements can also be incorporated to represent stiffeners on the skins or ribs.

In principle, this method has the great flexibility needed to allow a realistic representation of a typical structure which might have nonuniform frame, stiffeners or rib spacing, triangular ribs, nonuniform stiffening, etc. In practice, however, if it is attempted to represent the fine detail of a structure, a very large eigenvalue problem results which requires a very large computer for its solution. Often it will generally be more efficient to use the method in a qualitative sense to give an over-all picture of the response to noise and then use empirical modifications to estimate the stress response of small structural components.

The unreasonably high computer demands of the normal mode method are leading to the consideration of alternative methods for the determination of the response of structures to noise. One possibility is a more direct method of obtaining the response by direct integration of the equations of motion. New integration schemes are being developed which remain stable at longer time intervals than the ones traditionally related to half the shortest period of vibration. I think that this offers the most fruitful line of development of computer-based methods in the near future.

VI. Simplified Theory of Response

Although the above methods are being developed, there is a need for a method which can be used in design. Even when the more sophisticated methods are available, there will continue to be a need for a "back of the envelope" estimation procedure for initial design studies. Such a method will also be needed for empirical modifications to the more accurate over-all methods to estimate the effects of structural details such as cleats, cutouts, etc.

A simplified theory can be derived on the assumption that the major part of the response results from the contribution of one predominant mode. The tests on full-scale structures have shown that in certain types of structure (usually large skin plates) the response spectrum may only have one major peak resulting from one mode of vibration. In other structures, such as control surface skin plating, there may be many peaks in the response spectrum. Even in this case, however, the simple theory gives a reasonable estimate of the over-all stress level.

On the assumption that there is only one significant mode, i.e., the fundamental mode, contributing to the response and that the acoustic pressures are exactly in phase over the section of structure, a simplified expression for the rms stress in directly excited skin structure can be written as:

$$\langle\sigma^2(t)\rangle^{1/2} = \left|\frac{\pi}{2\eta} f_r G_p(f_r)\right|^{1/2}\sigma_o \qquad (5)$$

This expression was first derived from the consideration of a single degree-of-freedom system by Miles.[21] The natural frequency f_r and the unit stress parameter σ_o are derived from simplified models of the structure. For example, in the case of a stiffened skin, the individual plate members between the stiffeners are assumed to be single, isolated plates with fully fixed boundaries. This is obviously a very crude assumption, but gives reasonable estimates of the skin stresses in uniform thickness and stiffened plate structures.[22]

Modifications have been produced by Bayerdörfer[23] to allow for plate curvature. A design Nomograph[24] has been produced to facilitate use of this equation.

A comparison of estimates of skin stresses with measured results for several different structures, is shown in Fig. 8 taken from Ref. 24. It can be seen that the estimates are generally within a factor of 2 of the experimental results. The results outside this bound are mostly for curved plates.

However, in the case of integrally stiffened panels, the individual plates between stiffeners are generally narrower and the machined stiffeners themselves are less rigid than

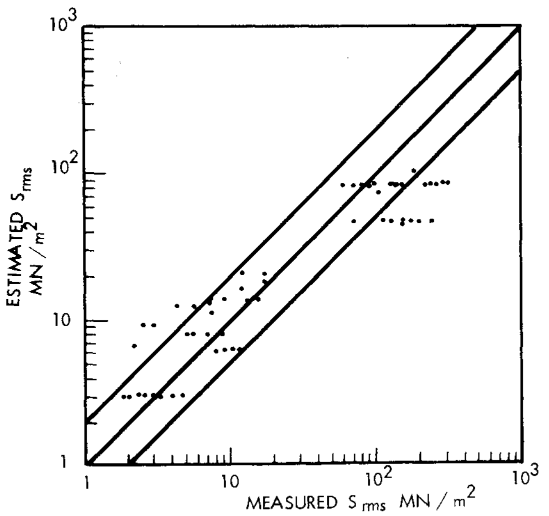

Fig. 8 Comparison of estimated and measured stresses in tailplane and control surface skins.

in the equivalent built-up structure. It is clear that in these cases, the fully fixed plate approximation would be considerably in error if used in estimations of the natural frequencies and also the stresses induced by acoustic loads. A better approximation in this case is to allow for the flexibility of the stiffeners supporting the sides of each plate in the panel.

In the case of control surfaces, two skins are attached together by ribs, and thus both skins and ribs vibrate because of the mechanical coupling between them. Even when the sound pressures are much greater on one side of the control surface, due to acoustic shielding, the stresses in both skins are very similar. These types of structures are not so amenable to the simple form of analysis [Eq. (5)] as now the energy incident on one skin, minus the reflected energy, is absorbed by two vibrating skins and the interconnecting ribs. Consequently, the stress level must be less than half and is possibly about one third of that which would have been induced in a single plate. The stresses in the surface plates can now be estimated as one third of the stress given by Eq. (5). This method has been found to give reasonable estimates of the skin stresses in control surfaces, but, of course, in its simplest form it does not give estimates of the rib stresses.

VII. Damping of High-Frequency Vibration

As the excitation has broadband frequency characteristics it is not possible to reduce the response by 'detuning' the structure. In some cases, for example, an increase in stiffness may increase the response because the generalized force may be greater at the increased frequency. In this situation, damping plays an important role and it may be that the most efficient structure is one in which the damping is increased artificially by the addition of energy absorbers.

In typical aerospace structures, the total damping comes from several energy-absorbing mechanisms.[25] The internal hysteresis in the metal itself usually provides an energy loss which is an order of magnitude below that from the other mechanisms. The damping resulting from radiation of sound and the energy absorption at joints may be major contributors. The acoustic radiation damping for flat and curved rectangular plates has been computed by Mead. Work on joint damping has been done by Mead and Eaton.[26] This latter work is rather empirical because the damping in joints has been found to be dependent on several parameters such as shear load, joint compression, amplitude, and time. In this work it has been shown that the joint damping can be increased significantly by the use of viscoelastic inserts.

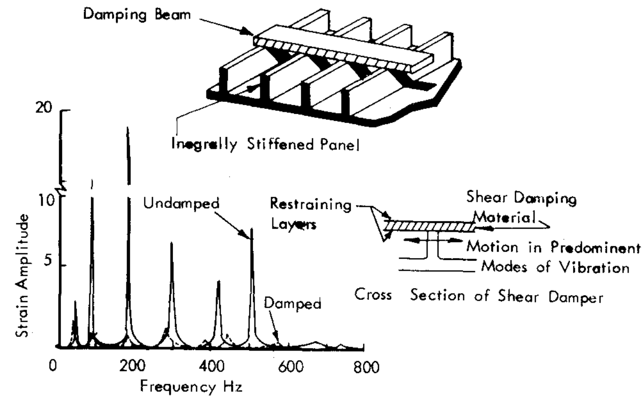

Fig. 9 The effect of a damping link on the response of an integrally stiffened panel.

The damping of conventional stiffened skin designs can be increased considerably by the use of viscoelastic sandwich material in place of the single metal sheet skin. In these designs, the viscoelastic material is sandwiched between two thin sheets of metal. This composite material can be formed and assembled using rivets in the normal way. Even greater damping can be achieved if the joints do not provide a rigid connection between the two face sheets of the sandwich. This can be done by bonding the inner face sheet to the support structure or by rivetting only the inner face to the support. These procedures may not be acceptable for primary structure which is designed to carry static loads. However, in limited regions where the acoustic loading may predominate (such as control surfaces, fillets, etc.) then it may be acceptable to produce an 'acoustic' design. A systematic study of the use of viscoelastic sandwich material has been made by Mead.[27]

In current European designs of aircraft there is widespread use of the integrally machined skin structures described in Sec. IV-A. These structures have fewer joints than conventional rivetted designs and hence the inherent damping in the structure is much lower. It has been shown by Clarkson and Cicci[28] that the damping of these structures can be increased very much by the addition of damping links across the tops of the stiffeners. Figure 9 illustrates this method and shows the reduction in response which is achieved over a wide frequency range.

VIII. Wave Propagation Theory

It has already been pointed out that very severe limitations are present in the normal mode approach because of the large number of modes which must be considered in the acoustic frequency range. There is also the added difficulty that if high damping elements are included in the structure, considerable coupling of the modes can occur. It is now difficult to identify normal modes either experimentally or theoretically. Alternative techniques based on the derivation of orthogonal modes have been suggested.

Faced with these difficulties, Mead[29] has developed a wave propagation theory which can be used to estimate the response of stiffened structures to traveling pressure fields. Although this was developed originally to study highly damped structures, it has been found to be equally applicable to lightly damped structures. The method applies to plates which are stiffened at regular intervals forming an assembly of identical structural units which are attached side by side. Closed-form analytical solutions have been derived for an infinitely long array of such elements. It has also been shown that an approximate solution can be derived for assemblies of a finite number of

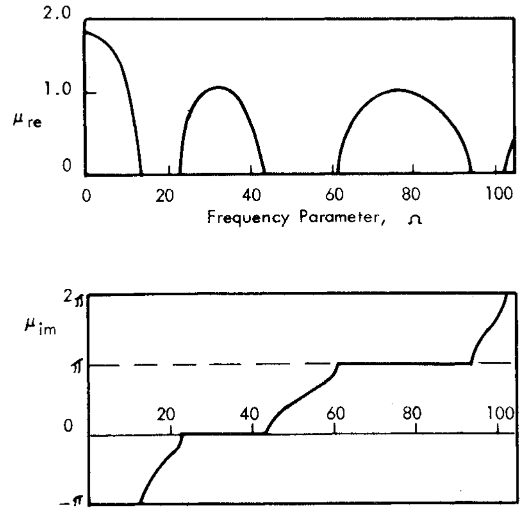

Fig. 10 Real and imaginary parts of the propagation constant for a multispan beam.

elements. The elements themselves can be plates and stiffeners as in a stiffened skin design, or boxes for a wing or stabilizer structure.

When a single-point harmonic force excites this simple structure, in certain frequency bands, the wave motion cannot spread out from the source without decaying in amplitude from one bay to the next. In the alternate frequency bands, flexural wave motion can propagate from bay to bay without decay (in the absence of damping), and each bay vibrates identically with its neighbor, apart from a characteristic phase difference, ϵ, known as the "propagation constant." This constant can be represented by real and imaginary parts μ_r, μ_i ($\epsilon = \mu_r + i\mu_i$) and typical values are shown in Fig. 10 for a beam on multiple supports. When μ_r is nonzero, unattenuated wave motion along the beam cannot occur; but when μ_r is zero, the motion (in the undamped case) is unattenuated along the beam. In this frequency range, a phase difference exists between adjacent bays (obtained from μ_i) and energy is propagated along the beam. The propagating flexural wave motion is not a simple sinusoidal wave, for such motion is necessarily interrupted and restricted at the supports. However, it can be analyzed into an infinite series of sinusoidal waves each of which travels at a different phase velocity. The wave should therefore be regarded as a wave group, the group velocity of which is the frequency times ϵ/length of the periodic element.

The response of a periodic beam to a convected harmonic pressure distribution (e.g., a propagating plane sound wave) has been calculated in a closed form.[30] In the absence of damping, an infinite response can occur in each propagation band, at the frequency at which the wave group velocity is equal to the convection velocity. The infinite response becomes finite as damping (acoustic or structural) is introduced into the system.

Now a random and diffuse sound field, or a convected boundary-layer pressure field may be decomposed into a spectrum of simple convected harmonic pressure distributions. The spectrum of the periodic beam response is related to this excitation through the beam response function corresponding to the simple harmonic pressure distribution. The beam response spectrum caused by a random pressure field can therefore be determined from a closed-form solution.[31]

Computer studies based on this method have been undertaken to investigate the response of a simple 'one-dimensional plate' to such pressure fields. The effects of beam damping, the rotational stiffness at the supports (representing the torsional stiffness of stringers on a plate) and the mean velocity of convection of a boundary-layer-type excitation have been included. Design optimisation studies have been made possible, whereby the structural configuration for minimum stress response may be found for a given excitation field.

Box-type structures under acoustic excitation are well suited to this method of analysis.[32] The flexural wave motion can travel in both top and bottom skins of the structure, the waves in both skins being coupled through the ribs. At the periodic support (i.e., rib location) of the skins, there are three freedoms: rotation of each skin and in-plane deflection of the rib. Three waves can therefore exist at each frequency. Neglecting the in-plane rib motion, the responses of the skins and ribs to homogeneous noise fields have been calculated from closed-form solutions, and the influence on the response of adding damping to either the ribs or skins has been investigated. In some frequency bands, the addition of damping to the ribs has been found to be detrimental.

In the case of the more practical finite periodic structure, the response may be greater at one end than at the other. Nevertheless, the total response of the finite structure may be regarded as the forced wave motion of the infinite structure, together with a set of free waves generated by the impingement of the forced wave motion on the extreme ends of the finite structure. Combined together, this total wave motion yields the exact response, the amplitude of which varies from bay to bay. Closed-form solutions to the response of finite periodic structures to random noise have thereby been obtained. The average responses of quite short finite structure (of only five bays) have been found to be surprisingly close to the responses of their infinite counterparts (even with light damping), especially when the noise field convects very rapidly over the structure. Recently this has been confirmed experimentally.

If the natural modes and frequencies of finite periodic structures are required, use may be made of the propagation constants of the system. For a system with only one degree of freedom at each periodic support, the natural modes consist of an equal and opposite pair of propagating waves which combine at a certain frequency to satisfy the extreme boundary conditions and to form a standing wave. At this frequency, the total phase shift associated with the wave traveling backwards and forwards once along the system, together with the phase shift due to its reflection at the extreme boundaries, is equal to an integral multiple of 2π. This concept has been developed to yield a very accurate method of finding natural frequencies of, say, continuous beams on rigid supports,[33] and has been used as the basis of Data Sheets for the estimation of natural frequencies of stiffened plates.[34]

The frequency equations for multibay finite periodic structures with two or more freedoms at the supports have been set up using these propagating wave concepts. Any type of extreme boundary conditions may be incorporated. In this case, the "total phase shift" concept cannot so obviously be used to determine the natural frequencies, but it has been found in fact to be applicable under quite a wide range of circumstances.

In current work about to be published, an 'energy method' of determining the curve of propagation constant vs frequency for general periodic systems (one, two, and multidimensional) has now been formulated. Abrahamson used the Hamiltonian approach and, using an appropriate approximate mode for the propagating wave, found the relationship between propagation constant and frequency for a rib-skin structure. Mead used a generalized coordinate, Lagrangian approach for a periodic system with multiple coupling between the elements and arrived at a similar relationship. In both cases, a Rayleigh-Ritz type of modification has been developed from the basic method.

Appropriate approximate wave forms are assumed for the motions in each element, and the phase difference between the motion at the ends of the element is assigned. The approximate frequency at which wave motion with this phase difference will occur is then obtained from an energy relationship, and this frequency is minimised as in the Rayleigh-Ritz method. A number of general theorems have also been enunciated and proved relating to wave motion in general multicoupled periodic systems.

IX. Conclusions

In such a brief review it has not been possible to describe adequately the contributions to the response of structures to noise which have been made over a period of about two decades. The majority of this period has seen the progressive development of the normal mode method and much work on individual components such as natural frequencies of a range of structural configurations. Recently it has been realized that further refinement of this technique may well lead to diminishing returns and that the most promising hope for the future may be in the development of the wave propagation method.

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